International Journal of Theoretical Physics, Vol. 34, No. 8, 1995

Pragmatic Versus Semantic Contextuality in Quantum Physics

Claudio Garola[1]

Received October 12, 1994

An approach to quantum physics (QP) is proposed that is characterized by the attempt to give up the verificationist theory of truth underlying the standard interpretation of QP. As a first step, an *observatively minimal* language L is constructed that is endowed with a Tarskian truth theory. Then, a set of axioms is stated by means of L that hold both in classical physics and in QP, and the further language L_e of all properties is constructed. The concepts of *meaning* and *testability* do not collapse in L and L_e, hence quantum logic is interpreted as a theory of testability in QP, and QP turns out to be semantically incomplete. Furthermore, semantic and pragmatic compatibility of physical properties are distinguished in L_e, and the concepts of testability and conjoint testability of statements are introduced. In this context some known quantum paradoxes can be avoided, and a new general principle (MGP) characterizes the truth mode of empirical physical laws. MGP invalidates the Bell theorem and, presumably, the Bell–Kochen–Specker theorem, and introduces a *pragmatic* contextuality in QP in place of the *semantic* contextuality that should occur because of these theorems.

1. INTRODUCTION

It is well known that the standard interpretation of quantum physics (QP) is based on the adoption of a verificationist theory of truth for the language of QP. This choice entails that no meaning can be attributed to nontestable statements, and implies that truth and epistemic accessibility (here, briefly, *testability*) be identified in the language of physics. But this identification can be criticized from an epistemological viewpoint (e.g., Popper, 1969) and constitutes in my opinion the deep root of many quantum paradoxes. Therefore, the question arises whether the language of QP can

[1] Dipartimento di Fisica dell'Universita', Lecce, Italy.

be endowed with a correspondence truth theory which allows us to restore the distinction between truth and testability.

I have tried to show in a number of papers (Garola, 1991, 1992a–c, 1993, 1994, 1995a,b) that the above question admits a positive answer. If correct, this answer is rather innovative. Indeed, contrary to Dummett (1975), I retain that a correspondence theory of truth, when formalized by a Tarskian theory of truth, does not imply any ontological assumption on reality, since the Tarskian theory is "ontologically neutral," as stated by Tarski himself (Tarski, 1952; Dalla Pozza and Garola, 1995). But a correspondence theory of truth, even if it is ontologically neutral, is compatible with some kind of realistic interpretation of QP, which conflicts with the orthodox position. Hence, the perspective proposed in the above papers is an alternative to the standard viewpoint, and I call it *semantic realism* (SR) in the following. I would like to report here on the most recent results obtained in this research, the details of which will be published in a forthcoming paper by myself and L. Solombrino.

2. THE LANGUAGE L

In order to realize the SR program I have constructed in the papers quoted in the Introduction a formalized language L (a first-order predicate calculus extended by means of *statistical quantifiers*) endowed with a Tarskian truth theory, by means of which all statements regarding testable physical properties of samples of a given physical system can be expressed. I have classified elsewhere L as an observative sublanguage of the higher order language L* that should be needed in order to express formally all laws of a physical theory (I will not discuss L* here, since its construction would be long and difficult). Actually, not all statements in L are observative, but L is *observatively minimal,* in the sense that it is a minimal sublanguage of L* that contains the observative part of L*. Since I cannot deal with L in detail for the sake of brevity, I report here only the basic scheme for constructing and interpreting it.

(i) *Alphabet of L.* The set X of individual variables; two disjoint sets $\mathcal{S}$ and $\mathcal{F}$ of monadic predicates, called *states* and *effects,* respectively; standard logical connectives $\neg$, $\wedge$ $\vee$, $\rightarrow$, $\leftrightarrow$, and quantifiers $\exists$, $\forall$; a family $\{\pi_r\}_{r\in[0,1]}$ of *statistical quantifiers*; the auxiliary symbols (.) and /.

(ii) *Formation rules.* The set Ψ of all well-formed formulas (wffs) of L is obtained by means of standard (recursive) formation rules, together with the following rule regarding statistical quantifiers:

$$\text{let } x \in X,\ A(x),\ B(x) \in \Psi,\ r \in [0,1]; \qquad \text{then } (\pi_r x)(A(x)/B(x)) \in \Psi$$

(iii) *Semantics.* The following sets and objects are introduced in L: the set I of *laboratories*; for every $i \in I$, the (finite) *domain* D_i of i; for every

$i \in I$, the set $\Sigma_i = \{\sigma_i: X \to D_i\}$ of the interpretations of the (individual) variables; for every $i \in I$, $\sigma_i \in \Sigma_i$, and $x \in X$, the *extension* $\sigma_i(x) \in D_i$ of x; for every $i \in I$ and $S \in \mathcal{S}$, the *extension* $\rho_i(S) \subseteq D_i$ of S; for every $i \in I$ and $F \in \mathcal{F}$, the *extension* $\rho_i(F) \subseteq D_i$ of F; the set $\hat{I} \subseteq I$ of all *statistically relevant* laboratories. Then, a Tarskian truth theory, suitably extended in such a way as to apply to statistical wffs, is assumed on L. Hence, the connectives $\neg$, $\wedge$, $\vee$, $\to$, $\leftrightarrow$ and the quantifiers $\exists$, $\forall$ are interpreted as *not, and, or, if . . . then, iff,* and *exists, for every,* respectively, as usual in classical logic. Furthermore, a statistical wff of the form $(\pi_r x)(A(x)/B(x))$ is true in the laboratory i iff, roughly speaking, the ratio between the number of elements in D_i that make $A(x)$ and $B(x)$ true and the number of elements that make $B(x)$ true (assumed to be nonzero in i) is r. Thus, whenever an interpretation σ_i of the variables is given in the laboratory i, every wff of L has a truth value in i.

(iv) *Interpretation.* Following Ludwig (1983), states and effects are (bijectively) interpreted on equivalence classes of the sets Π and $\mathcal{R}$ of all preparing and dichotomic registering devices associated to a given physical system $\mathfrak{S}$, respectively. Laboratories are interpreted as space-time regions in the actual world. For every $i \in I$, the set D_i is interpreted as the set of all individual samples of $\mathfrak{S}$ prepared in i, or *physical objects,* hence for every $\sigma_i \in \Sigma_i$ and $x \in X$, $\sigma_i(x)$ is a physical object in D_i. For every $i \in I$, the extension $\rho_i(S)$ of the state S in the laboratory i is interpreted as the set of all physical objects prepared in i by means of devices belonging to the equivalence class S, while the extension $\rho_i(F)$ of the effect F consists of all physical objects in i which *would pass* the test whenever tested with any device belonging to the equivalence class F. Finally, $\hat{I}$ is intuitively interpreted as the set of laboratories where a large number of physical objects is produced and all preparations and registrations are performed with the caution required by the physical theory that is adopted (the Tarskian truth theory defined on L refers to $\hat{I}$ rather than to I; in particular, this occurs when universally quantifying on laboratories, as in the expression "for every laboratory i").

(v) By referring to $\hat{I}$, three preorder relations can be defined on Ψ and $\mathcal{F}$.

Logical preorder $\subset$:

for every $A_1, A_2 \in \Psi$, $A_1 \subset A_2$ iff for every $i \in \hat{I}$, A_2 is true for every interpretation $\sigma_i \in \Sigma_i$ such that A_1 is true;

for every $x \in X$ and $F_1, F_2 \in \mathcal{F}$, $F_1 \subset F_2$ iff $F_1(x) \subset F_2(x)$.

Statistical preorder $\angle$:

for every $A_1, A_2 \in \Psi$, $A_1 \angle A_2$ iff for every $S \in \mathcal{S}$ and $i \in \hat{I}$, $(\pi_{r_1}x)(A_1/S(x))$ and $(\pi_{r_2}x)(A_2/S(x))$ true in i imply $r_1 \leq r_2$;

for every $x \in X$ and $F_1, F_2 \in \mathcal{F}$, $F_1 \angle F_2$ iff $F_1(x) \angle F_2(x)$.

Deterministic preorder $<$:

for every $A_1, A_2 \in \Psi$, $A_1 < A_2$ iff for every $S \in \mathcal{S}$, $(\forall x)(S(x) \to A_1)$ true in every $i \in \hat{I}$ implies $(\forall x)(S(x) \to A_2)$ true in every $i \in \hat{I}$;

for every $x \in X$ and $F_1, F_2 \in \mathcal{F}$, $F_1 < F_2$ iff $F_1(x) < F_2(x)$.

The above preorders canonically induce three equivalence relations $\equiv$, $\simeq$, $\approx$ on Ψ and $\mathcal{F}$, which we call logical, statistical, and deterministic equivalence relations, respectively (note that the statistical and the deterministic preorders on $\mathcal{F}$ translate in our context the orders introduced in the Ludwig and Piron approaches to the foundations of QP, respectively).

3. STATES AND EFFECTS

Based on the interpretation of L discussed in Section 2, I have introduced in a previous paper (Garola, 1991) a set of assumptions and definitions on states and effects that hold both in QP and in classical physics (CP). But a deeper analysis shows that this conceptual apparatus is not adequate in the case of compound quantum systems. Therefore, it will be substituted here by a new generalized set of assumptions and definitions, which reduces to the previous one in the case of noncompound systems [the symbol $n(\phi)$ denotes the number of elements in the finite set ϕ in the following].

AX 1. (i) Let $S_1, S_2 \in \mathcal{S}$. If, for every $i \in \hat{I}$ and $F \in \mathcal{F}$,

$$n(\rho_i(S_1) \cap \rho_i(F)) \cdot n(\rho_i(S_2)) = n(\rho_i(S_2) \cap \rho_i(F)) \cdot n(\rho_i(S_1))$$

then $S_1 = S_2$.

(ii) For every $S_1, S_2 \in \mathcal{S}$, if $S_1 \neq S_2$, then, for every $i \in \hat{I}$, $\rho_i(S_1) \cap \rho_i(S_2) = \emptyset$.

AX 2. For every $F_1, F_2 \in \mathcal{F}$, $F_1 = F_2$ iff $F_1 \equiv F_2$ iff $F_1 \simeq F_2$.

AX 3. For every $F \in \mathcal{F}$, an $F' \in \mathcal{F}$ exists such that, for every $i \in \hat{I}$, $\rho_i(F') = D_i \setminus \rho_i(F)$.

DEF 1. For every $S \in \mathcal{S}$, the set $\mathcal{F}_S = \{F \in \mathcal{F} \mid \text{for every } i \in \hat{I},\ \rho_i(S) \subseteq \rho_i(F)\}$ is called the *certainly true domain* of S in $\mathcal{F}$.

DEF 2. For every $i \in \hat{I}$ and $S \in \mathcal{S}$, $\hat{\rho}_i(S) = \cap_{F \in \mathcal{F}_S} \rho_i(F)$.

DEF 3. The set $\mathcal{S}_P = \{S \in \mathcal{S} \mid \text{for every } S^* \in \mathcal{S},\ \rho_i(S^*) \subseteq \hat{\rho}_i(S)$ in every $i \in \hat{I}$ implies $S^* = S\}$ is called the set of *pure states* of the physical system.

AX 4. For every $i \in \hat{I}$ and $S_1, S_2 \in \mathcal{S}_P$,

$$n(\rho_i(S_1) \cap \hat{\rho}_i(S_2)) \cdot n(\rho_i(S_2)) = n(\rho_i(S_2) \cap \hat{\rho}_i(S_1)) \cdot n(\rho_i(S_1))$$

DEF 4. The symbol $\perp\!\!\perp$ denotes the preclusivity (nonreflexive and symmetric) relation on $\mathcal{S}_P$ such that, for every $S_1, S_2 \in \mathcal{S}_P$, $S_1 \perp\!\!\perp S_2$ iff, for every $i \in \hat{I}$, $\rho_i(S_1) \cap \hat{\rho}_i(S_2) = \emptyset$ [equivalently, $\rho_i(S_2) \cap \hat{\rho}_i(S_1) = \emptyset$].

DEF 5. The symbol $^{\perp}$ denotes the weak orthocomplementation on the power set $\mathcal{P}(\mathcal{S}_P)$ such that

$$H \in \mathcal{P}(\mathcal{S}_P) \rightarrow H^{\perp} = \{S \in \mathcal{S}_P \mid \text{for every } S^* \in H,\ S \perp\!\!\perp S^*\}$$

DEF 6. The symbol $^{\perp\perp}$ denotes the closure operation on the power set $\mathcal{P}(\mathcal{S}_P)$ such that $H \in \mathcal{P}(\mathcal{S}_P) \rightarrow (H^{\perp})^{\perp} \in \mathcal{P}(\mathcal{S}_P)$.

DEF 7. The symbol $(\mathcal{L}, \subseteq)$ denotes the complete orthocomplemented lattice of all $^{\perp\perp}$-closed subsets of the power set $\mathcal{P}(\mathcal{S}_P)$, ordered by set-theoretic inclusion [then $\cap$ and $\cup$ denote meet and join in $(\mathcal{L}, \subseteq)$, respectively, in the following].

DEF 8. Let $\mathcal{S}_t$: $F \in \mathcal{F} \rightarrow \mathcal{S}_t(F) = \{S \in \mathcal{S}$: for every $i \in \hat{I}$, $\rho_i(S) \subseteq \rho_i(F)\} \in \mathcal{P}(\mathcal{S}_P)$. Then, $\mathcal{S}_t(F)$ is called the *certainly-yes domain* of F. Analogously, the set $\mathcal{S}_f(F) = \{S \in \mathcal{F}$: for every $i \in \hat{I}$, $\rho_i(S) \cap \rho_i(F) = \emptyset\}$ is called the *certainly-no domain* of F.

DEF 9. The set $\mathcal{F}_e = \{F \in \mathcal{F} \mid \mathcal{S}_t(F) \in \mathcal{L},\ \mathcal{S}_f(F) = \mathcal{S}_t^{\perp}(F)\} \subseteq \mathcal{F}$ is called the set of (nouns of) *observative exact effects*, or *testable properties*.

AX 5. For every $F_1, F_2 \in \mathcal{F}_e$, $\mathcal{S}_t(F_1) \subseteq \mathcal{S}_t(F_2)$ implies $F_1 \subset F_2$.

AX 6. The poset $(\mathcal{S}_t(\mathcal{F}_e), \subseteq)$ is dense in $(\mathcal{L}, \subseteq)$.

DEF 10. The symbol $(\mathcal{E}_e, \subset)$ denotes the completion by cuts of $(\mathcal{F}_e, \subset)$ (a complete orthocomplemented lattice isomorphic to $(\mathcal{L}, \subseteq)$); the set $\mathcal{E}_e$ is called the set of the *exact effects*, and the set $\mathcal{D}_e = \mathcal{E}_e \backslash \mathcal{F}_e$ is called the set of the *theoretical exact effects*, or *theoretical properties*.

AX 7. The lattice $(\mathcal{L}, \subseteq)$ is atomic, and $\{\{S\} \mid S \in \mathcal{S}_P\}$ is the set of its atoms.

DEF 11. Let us still denote by $\mathcal{S}_t$ the canonical extension of the mapping $\mathcal{S}_{t1}$ defined on $\mathcal{F}_e$, to $\mathcal{E}_e$. Then, for every $S \in \mathcal{S}_P$, the exact effect $E_S = \mathcal{S}_t^{-1}(\{S\})$, which is an atom of $(\mathcal{E}_e, \subset)$, is called the *support* of S. Furthermore, S is said to be a *first-type* state iff $E_S \in \mathcal{F}_e$, a *second-type* state iff $E_S \in \mathcal{D}_e$.

AX 8. For every $S \in \mathcal{S}_P$, S is a first-type state iff an effect $F_S \in \mathcal{F}$ exists such that, for every $i \in \hat{I}$, $\hat{\rho}_i(S) = \rho_i(F_S)$, and $F_S = E_S = \mathcal{S}_t^{-1}(\{S\})$ in this case.

I cannot comment on the above set of assumptions and definitions in detail here, and I limit myself to pointing out some features that are relevant in order to understand their meaning intuitively. Axioms 1 and 2 implicitly define the classes of preparing and registering devices on which states and effects are interpreted. Axiom 3 introduces a complement F' for every effect $F \in \mathcal{F}$. Axiom 4 introduces a symmetry property on the set $\mathcal{S}_P$ of *pure states* that is basic for defining the *closure* operation $^{\perp\perp}$ on the power set $\mathcal{P}(\mathcal{S}_P)$, by means of which one can introduce the complete orthocomplemented lattice $(\mathcal{L}, \subseteq)$ of all closed subsets of $\mathcal{P}(\mathcal{S}_P)$. Then, one can select a subset $\mathcal{F}_e$ of *observative exact effects* in the set $\mathcal{F}$ of all effects that can be interpreted as the set of all *testable physical properties* of the physical system. Axiom 5 implies that the order relations $\subset$, $\angle$, $<$ defined on $\mathcal{F}$ can be identified on $\mathcal{F}_e \subseteq \mathcal{F}$. Axiom 6 implies that the standard completion by cuts (Garola, 1985) ($\mathcal{E}_e$, $\subset$) of the poset ($\mathcal{F}_e$, $\subset$) is a lattice isomorphic to ($\mathcal{L}$, $\subseteq$), but it does not assume that ($\mathcal{F}_e$, $\subset$) itself is a lattice. Thus, one can introduce a new set $\mathcal{D}_e = \mathcal{E}_e \backslash \mathcal{F}_e$ of *theoretical properties,* that are born whenever the completion procedure is performed: of course, $\mathcal{D}_e$ can be void, as occurs, for instance, in CP. By introducing Axiom 7 one can associate a property E_S (the *support* of S) with every pure state S, and distinguish between *first-type* pure states (whose supports belong to $\mathcal{F}_e$) and *second-type* pure states (whose supports belong to $\mathcal{D}_e$). Then Axiom 8 implies that, whenever S is a first-type pure state, E_S is the unique minimal property that is certainly true in every laboratory i for every physical object in the state S. But it is important to note that, even if E_S characterizes S for every pure state S, the extension $\rho_i(S)$ of S in every laboratory i is strictly contained in the extension $\rho_i(E_S)$ of the support of S in QP, while $\rho_i(S) = \rho_i(E_S)$ in CP. Therefore, S and E_s cannot be identified from a semantic viewpoint in QP, which is important for the solution of some old EPR-like paradoxes.

The above system of axioms implies that the lattice ($\mathcal{E}_e$, $\subset$) of all properties is complete, orthocomplemented, and atomic, like the lattice of questions in the Mackey (1963) approach and the lattice of propositions in the Jauch (1968) and Piron (1976) approaches. But the distinction between theoretical and testable properties does not appear in these approaches, while it is basic in my opinion for solving some problems in the quantum theory of compound physical systems. For instance, it leads to overthrowing the conclusion by Aerts (1982), who assumes that testable properties exist which are associated to second-type states that are not ruled out by some superselection rule, and then concludes that compound quantum systems generally are nonseparable. Indeed, in the SR approach separate quantum systems can exist, since the properties associated to second-type states are considered as mere mathematical symbols that are introduced in order to transform a poset into a lattice. This viewpoint is supported by the fact that second-type pure states can be distinguished from mixed states in Hilbert space quantum theory (HSQT) only by means of correlation properties, that is, second-order properties (roughly speaking, properties of properties), and it would be wrong to attribute to these the same logical status attributed to the first-order testable properties that appear in the language L and that are represented by projections in HSQT.

Finally, I note that the above system of axioms is not complete from several viewpoints. For instance, it does not take into account mixed states, nor does it allow one to distinguish between CP and QP, since further axioms should be needed in order to make $(\mathscr{E}_e, \subset)$ distributive (CP), or orthomodular and satisfying the covering law (QP). But the axioms listed above are sufficient for the aims of the present paper.

4. THE PROPERTIES LANGUAGE L_e

The set $\mathscr{E}_e = \mathscr{F}_e \cup \mathscr{D}_e$ of testable and theoretical properties in the framework constructed in Section 3 naturally leads us to introduce a new language L_e that is identical to L, with the exception of the set $\mathscr{F}$, which is substituted by $\mathscr{E}_e$ (hence we call L_e the *properties language* in the following). This new language will still be endowed with a Tarskian truth theory: but the extensions of theoretical predicates have no empirical interpretation and are defined conventionally.

It must be noted that L_e, not L, is the language that must be taken into account if one wants to compare the SR approach to QP with other approaches: for, the poset $(\mathscr{F}, \subset)$ of the effects in L generally is not a lattice, while the poset $(\mathscr{E}_e, \subset)$ of all testable and theoretical properties is endowed with a lattice structure that one can compare with the lattice structures appearing in other approaches (Section 3). But it must then be stressed that the attribution of truth values to sentences in the new language L_e may be conventional, not only in the case of molecular sentences (as in L, because of the use of the formal rules of classical logic), but also in the case of atomic sentences of the form $E(x)$, whenever E is a theoretical property that has no direct physical interpretation.

5. CONSISTENCY, COMPATIBILITY, AND TESTABILITY

The theoretical apparatus described in the previous sections and, in particular, the adoption of a Tarskian truth theory in L_e, allows one to introduce some conceptual distinctions that are impossible whenever a verificationist truth theory is adopted. In particular, one can distinguish between semantic and pragmatic compatibility of properties and between testability and truth of sentences. I will discuss these distinctions here rather naively, in order to avoid technicalities.

(i) Let S_1 and S_2 be pure states. Intuitively, one can say that S_1 and S_2 are semantically compatible, or consistent, iff the set of all properties that are certainly true whenever a physical object x is in the state S_1 does not contain the negation of any property that is certainly true whenever a physical object is in the state S_2 (in other words, the information embodied in S_1 does not conflict with the information embodied in S_2). This intuitive notion can be formalized in our context by introducing, for every $S \in \mathscr{S}_P$, the *certainly-true domain* $\mathscr{E}_S = \{E \in \mathscr{E}_e | E_S \subset E\}$ of S in $\mathscr{E}_e$ [equivalently, $\mathscr{E}_S = \{E \in$

$\mathscr{E}_e \mid$ for every $i \in \hat{I}$, $\rho_i(S) \subseteq \rho_i(E)\}]$ and the *certainly-false domain* $\mathscr{E}_S^\perp = \{E \in \mathscr{E}_e \mid E^\perp \in \mathscr{E}_S\}$ of S in $\mathscr{E}_e$ [equivalently, $\mathscr{E}_S^\perp = \{E \in \mathscr{E}_e \mid$ for every $i \in \hat{I}$, $\rho_i(S) \cap \rho_i(E) = \varnothing\}]$. Indeed, one says that S_1 and S_2 are *semantically compatible*, or *consistent*, and writes $S_1\, C\, S_2$, iff $\mathscr{E}_{S_1} \cap \mathscr{E}_{S_2}^\perp = \varnothing = \mathscr{E}_{S_1}^\perp \cap \mathscr{E}_{S_2}$. One can then prove (Garola, 1992a) that C is an accessibility relation (it is reflexive and symmetric but not, generally, transitive) and that $S_1\, C\, S_2$ in QP iff $\langle \psi_1 \mid \psi_2 \rangle \neq 0$, $\mid \psi_1 \rangle$ and $\mid \psi_2 \rangle$ being the vectors that represent S_1 and S_2, respectively, in HSQT.

Let us come to $\mathscr{E}_e$. Let E_1, E_2 be (testable or theoretical) properties. Intuitively, one can say that E_1 and E_2 are semantically compatible, or consistent, iff in some laboratory i some physical objects exist that share both E_1 and E_2. This intuitive notion can be formalized in our context. Indeed, one says that E_1 and E_2 are *semantically compatible*, or *consistent*, and writes $E_1\, C\, E_2$, iff an $i \in \hat{I}$ exists such that $\rho_i(E_1) \cap \rho_i(E_2) \neq \varnothing$. One can then prove that, whenever E_1 and E_2 are the supports of the pure states S_1 and S_2, respectively, $E_1\, C\, E_2$ iff $S_1\, C\, S_2$.

(ii) Let F_1, F_2 be testable properties. Intuitively, one can say that F_1 and F_2 are pragmatically compatible iff they are compatible according to the standard notion adopted in QP, that is, iff one can establish whether they both hold for a physical object x by means of a suitable measurement on x. This intuitive notion can be formalized in our context. Indeed, one says that F_1 and F_2 are *pragmatically compatible*, or *conjointly testable*, or simply *compatible*, and writes $F_1\, K\, F_2$, iff a testable property $F \in \mathscr{F}_e$ exists such that, for every laboratory $i \in \hat{I}$, $\rho_i(F) = \rho_i(F_1) \cap \rho_i(F_2)$. Then, the relation K can be characterized by showing that $F_1\, K\, F_2$ iff the meet $F_1 \cap F_2$ in the lattice $(\mathscr{E}_e, \subset)$ belongs to $\mathscr{F}_e$ and is such that, for every $i \in \hat{I}$, $\rho_i(F_1 \cap F_2) = \rho_i(F_1) \cap \rho_i(F_2)$.

Both the definition of K and its characterization can be extended to the case of n testable properties. Moreover, the empirical interpretation of pragmatic compatibility suggests that we introduce two further axioms that are needed in order to deal with this kind of compatibility.

AX 9. For every $F_1, F_2 \in \mathscr{F}_e$, $F_1\, K\, F_2$ iff $F_1\, K\, F_2^\perp$ iff $F_1^\perp\, K\, F_2$ iff $F_1^\perp\, K\, F_2^\perp$.

AX 10. Let $F_1, F_2, \ldots, F_n \in \mathscr{F}_e$. Then, $F_1, F_2, \ldots, F_n$ are (pragmatically) compatible whenever they are pairwise compatible, that is, $F_j\, K\, F_k$ for every $j, k = 1, 2, \ldots, n$.

(iii) The notions of testability and conjoint testability defined on $\mathscr{E}_e$ can be canonically extended to the set Ψ_e of all sentences of the language L_e. Indeed, one first says that a sentence A of L_e is *testable* whenever, in every laboratory $i \in \hat{I}$, the truth value of A for every interpretation of the (individual) variables can be determined by means of suitable measurements. By using this definition one can easily select some sample subsets of testable formulas of L_e [for instance, the set $\mathcal{F}_e(x) = \{F(x) \mid F \in \mathcal{F}_e\}$] and prove an important *criterion of testability,* which states that a molecular open wff $A(x)$ of L_e is testable iff it is semantically equivalent to an atomic wff $F_A(x)$, with $F_A \in \mathcal{F}_e$. Then, one defines *conjoint testability* in Ψ_e by saying that the wffs A_1, A_2 of L_e are *conjointly testable* iff A_1 and A_2 are testable and, for every laboratory $i \in \hat{I}$ and interpretation of the variables $\sigma_i \in \Sigma_i$, the truth values of A_1 and A_2 can be determined conjointly by means of suitable measurements. Then, one can show that, if the wffs $A_1(x)$, $A_2(x)$, . . . , $A_n(x)$ are testable, they are also conjointly testable iff the wff $A_1(x) \wedge A_2(x) \wedge \ldots \wedge A_n(x)$ is testable.

The above definitions and results are important from several viewpoints. Indeed, by suitably selecting subsets of testable wffs of the classical language L_e, one can recover quantum logic as a *theory of testability* in QP rather that an alternative to classical logic [see Garola (1991); some changes in the treatment should now be made in order to take into account the present generalization of the system of axioms]. Analogously, fuzzy logic can be recovered in an extended classical framework. Furthermore, the problem of the completeness of QP can be restated and refined by making reference to the language L_e and distinguishing between *s-completeness* (that is, completeness of the theory with respect to all interpreted wffs of L_e) and *t-completeness* (that is, completeness of the theory with respect to all testable interpreted wffs of L_e); then, it is possible to show that QP is t-incomplete, hence s-incomplete (Garola, 1992a). Finally, ideal quantum measurements can be examined in an SR context and shown to supply values that can be attributed to the observables before the measurements and independently of the measurements themselves; this obviously contradicts the orthodox interpretation, according to which there are physical properties, or values of observables, that are actualized by the measurements and cannot be assigned independently of the measurements themselves.

6. PRAGMATIC VERSUS SEMANTIC CONTEXTUALITY IN QP

The last remark at the end of Section 5 shows explicitly that SR provides a noncontextual interpretation of QP, even if it carefully avoids any ontological assumption on reality. This is unacceptable from an orthodox viewpoint, and physicists usually think that a noncontextual interpretation is proven to be impossible by the Bell–Kochen–Specker (Bell-KS) theorem and by the Bell theorem (Mermin, 1993). In order to vindicate SR, I have defended (Garola, 1994, 1995a,b) the following thesis: (i) the Bell-KS and Bell theorems stand on an implicit assumption (metatheoretical classical principle, or MCP), that is, the assumption that any *empirical* quantum physical law holds in every laboratory, even in physical situations that are not observative in the sense that QP prohibits checking whether they actually occur; (ii) MCP is not consistent with the basic operational philosophy of QP, and a generalization of it is required, which reduces to MCP in CP but takes into account the existence of sentences that are not conjointly testable in the language of more sophisticated theories; (iii) if this generalization is provided, the Bell theorem and, I maintain, the Bell-KS theorem can be invalidated, hence a noncontextual interpretation of QP is not prohibited by QP itself.

Let us discuss the above thesis in more detail. To this end, let us consider within our present framework the distinction between theoretical and empirical physical laws, which is standard in epistemology [received viewpoint, e.g., Carnap (1966)]. Theoretical laws contain primitive or derived theoretical terms and their formal statement requires the general language L* (Section 2) that should admit quantification on predicative variables: but some theoretical laws can also be expressed by means of the smaller languages L and L_e (L_e contains, in particular, primitive theoretical properties), and in this case the sentences stating theoretical laws are nontestable (hence they have a conventional truth value only). On the contrary, empirical laws can be expressed by means of testable sentences of L, or L_e (they are usually deduced from theoretical laws).

Let us briefly explore the form taken by those (theoretical or empirical) physical laws that can be expressed by means of L_e. Bearing in mind our interpretation of L_e, a typical sample of physical law is a quantified sentence of the form

$$V = (\pi_r x)(A(x)/S(x))$$

with $r \in [0, 1]$, $S \in \mathcal{S}$, and $A(x)$ an open molecular wff of L_e where only predicates denoting properties occur. Indeed, V prescribes the percentages of objects in the state S for which the sentence $A(x)$ regarding properties of physical objects is true (more complex forms of laws are not excluded, but do not interest us here).

The wff V expresses an empirical physical law whenever $A(x)$ is testable; hence, because of the criterion of testability (Section 5), whenever a testable property $F_A \in \mathcal{F}_e$ exists such that $A(x) \equiv F_A(x)$; in this case $V \equiv (\pi_r x)(F_A(x)/S(x))$, and the truth value of V can be determined empirically in every laboratory $i \in \hat{I}$ by means of any registering device in the class denoted by F_A. Whenever $A(x)$ is nontestable, that is, whenever no $F_A \in \mathcal{F}_e$ exists such that $A(x) \equiv F_A(x)$, V expresses a theoretical physical law, and its truth value in a laboratory i, though defined in our approach, cannot be directly tested; furthermore, it is partially conventional, because of the conventions introduced for logical connectives and/or because of the possible occurrence of primitive theoretical predicates.

An empirical physical law, which one can assume to be formally deduced from a theoretical law, is usually associated, in a given laboratory *i*, with a set of statements (which can be void) that express properties of physical objects that hold in the laboratory *i*, or *premises*. The law and the premises, together with the assignment of the *boundary conditions*, should allow a physicist to state predictions on further properties of the physical objects. In our framework, the premises are statements of L_e, and the boundary conditions consist in the assignment of the truth value of a wff of the form $S(x)$. Then, one can reformulate MCP by saying that it states that an empirical physical law is true in every laboratory independent of the (pragmatic) compatibility of the premises. This reformulation clearly shows that MCP is inconsistent with the basic operational philosophy of QP, which would rather suggest that an empirical physical law cannot be asserted to be true in physical situations that are not epistemically accessible, as occurs whenever noncompatible premises are assumed (of course, one can neither say that the law is necessarily false in these cases). This critique suggests that we substitute MCP with the following weaker *metatheoretical generalized principle.*

MGP. Let $V \in \Psi_e$ express a theoretical physical law, let $x \in X$, $S \in \mathcal{S}$, $A(x) \in \Psi_e$, let $A(x)$ be testable, and let the wff $V_A = (\pi_r x)(A(x)/S(x))$ express an empirical physical law deduced from V. Then V_A can be asserted to be true in every laboratory $i \in \hat{I}$ where a set of conjointly testable premises is assumed.

The restricted availability of empirical physical laws stated by MGP has far-reaching consequences. Indeed, Bell's theorem can be invalidated whenever MGP is accepted together with the distinction between empirical and theoretical properties established in Section 3 (Garola, 1994, 1995a,b). Hence, locality (equivalently, noncontextuality for separated physical systems) can be reconciled with QP. Furthermore, I maintain that MGP also invalidates the Bell-KS theorem (I am presently working on this topic). If the last conjecture is accepted, even unrestricted noncontextuality is reconciled with QP, and no further objection based on Bell-type arguments can be raised against SR.

It must, however, be noted that MGP introduces a new kind of contextuality, which refers to the validity of empirical physical laws rather than to the attribution of truth values to observative statements. I will call this kind of contextuality *pragmatic,* differentiating it from the conventional kind of contextuality (or noncontextuality), which will be called *semantic* here. Table I synthesizes the differences between the conventional and the SR approaches.

Table I.

	Semantic contextuality	Semantic contextuality for separated systems (nonlocality)	Pragmatic contextuality
Canonical viewpoint	Yes (Bell-KS theorem)	Yes (Bell theorem)	No
SR viewpoint	No (SR+MGP)	No (SR+MGP)	Yes

7. THE CLASSICAL EPR-LIKE PARADOXES

I would like to close my synthesis of the SR approach by observing that it provides an original and straightforward answer to the old arguments by Furry (1936a,b) and Bohm and Aharonov (1957), who aimed to show that paradoxes follow from the quantum description of the Einstein–Podolsky–Rosen (EPR) thought-experiment (note that the SR confutation does not require the use of MGP).

(i) *The Furry argument.* In brief, this argument consists in considering a two-particle system in a suitable second-type (pure) state, say S, in making a measurement on particle 1 when it is far apart from particle 2 and, finally, in deducing the state, say S_2, of particle 2 after the measurement by means of standard QP rules. Since no interaction occurs with 2, Furry deduces that S_2 is the state of 2 even before the measurement. But this result implies that the whole system should have been described by a mixture of pure first-type states rather than by the pure second-type state S, which is obviously a paradox.

The Furry reasoning immediately proves to be incorrect from the viewpoint of SR: indeed, according to SR, the test performed on 1 allows one to attribute a property to 2 which is the support of the state S_2, not to say that 2 was in the state S_2 before the measurement.

One could still object that the state of 2 is S_2, in any case, after the measurement of 1, so that, even in the SR explanation, a change occurs at 2 which is not justified by a local interaction. Then, I note that this change is interpreted in SR as a refinement of the information on the sample of 2 that belongs to the sample x of the system on which the measurement is done. This refinement can occur without any change of the testable physical properties of (the sample of) 2. More specifically, it can be proved that a measurement on 1 enlarges the set of properties of 2 that are known to be true (equivalently, the set of properties that are certainly true for 2); yet, the set $\mathscr{E}^{T}_{x2}$ of all properties that are true for the given sample of 2 does not change, nor it changes the set $\mathscr{E}^{F}_{x2}$ of all properties of the sample that are false (of course, $\mathscr{E}^{T}_{x2}$ and $\mathscr{E}^{F}_{x2}$ depend on the specific sample that is being considered).

(ii) *The Bohm–Aharonov argument.* This argument can be summarized as follows. Let us consider the same physical situation considered above. Then, one can attribute different properties to the physical object 2, depending on the choice of the measurement that is performed on 1. But these properties are generally noncompatible. This means, according to the standard interpretation of QP, that the actual properties of 2 are determined by an arbitrary choice made by a faraway observer, who does not interact in any way with the physical object 2. This introduces some kind of subjectivity in QP, which sounds paradoxical.

The answer of SR to this reasoning is immediate. According to SR, the properties of particle 2 considered in the Bohm–Aharanov argument are pragmatically noncompatible, but semantically compatible (consistent). Nothing prohibits that they be conjointly true for a sample of particle 2, both before and after a measurement of 1, even if testing one of them prohibits testing the other one. The paradox in the orthodox interpretation follows from the identification of truth (and meaning) with epistemic accessibility, which leads to identifying pragmatic and semantic compatibility. Hence, ultimately, the paradox follows from the adoption of a verificationist theory of truth, and it is removed whenever this theory is given up.

REFERENCES

Aerts, D. (1982). Description of many physical entities without the paradoxes encountered in quantum mechanics, *Foundations of Physics,* **12,** 1131.

Bohm, D., and Aharonov, Y. (1957). Discussion of experimental proofs for the paradox of Einstein, Rosen, and Podolski, *Physical Review,* **108,** 1070.

Carnap, R. (1966). *Philosophical Foundations of Physics,* Basic Books, New York.

Dalla Pozza, C., and Garola, C. (1995). A pragmatical interpretation of intuitionistic propositional logic, *Erkenntnis,* to appear.

Dummett, M. (1975). What is a theory of meaning? I, in *Mind and Language,* S. D. Guttenplan, ed., Oxford University Press, Oxford.

Furry, W. H. (1936a). Note on the quantum-mechanical theory of measurement, *Physical Review,* **49,** 393.

Furry, W. H. (1936b). Remarks on measurements in quantum theory, *Physical Review,* **49,** 476.

Garola, C. (1985). Embedding of posets into lattices in quantum logic, *International Journal of Theoretical Physics,* **24,** 423.

Garola, C. (1991). Classical foundations of quantum logic, *International Journal of Theoretical Physics,* **30,** 1.

Garola, C. (1992a). Semantic incompleteness of quantum physics, *International Journal of Theoretical Physics,* **31,** 809.

Garola, C. (1992b). Quantum logics seen as quantum testability theories, *International Journal of Theoretical Physics,* **31,** 1639.

Garola, C. (1992c). Truth versus testability in quantum logic, *Erkenntnis,* **37,** 197.

Garola, C. (1993). Semantic incompleteness of quantum physics and EPR-like paradoxes, *International Journal of Theoretical Physics,* **32,** 1863.

Garola, C. (1994). Reconciling local realism and quantum physics: A critique to Bell, *Teoreticheskaya i Matematicheskaya Fizika,* **99,** 285.

Garola, C. (1995a). Criticizing Bell: Local realism and quantum physics reconciled, *International Journal of Theoretical Physics,* **34,** 253.

Garola, C. (1995b). Questioning nonlocality: An operational critique to Bell's theorem, in *The Foundations of Quantum Mechanics. Historical Analysis and Open Questions,* C. Garola and A. Rossi, eds., Kluwer, Dordrecht.

Jauch, J. M. (1968). *Foundations of Quantum Mechanics,* Addison-Wesley, Reading, Massachusetts.

Ludwig, G. (1983). *Foundations of Quantum Mechanics I,* Springer-Verlag, New York.

Mackey, G. W. (1963). *The Mathematical Foundations of Quantum Mechanics,* Benjamin, New York.

Mermin, N. D. (1993). Hidden variables and the two theorems of John Bell, *Reviews of Modern Physics,* **65,** 803.

Piron, C. (1976). *Foundations of Quantum Physics,* Benjamin, Reading, Massachusetts.

Popper, K. R. (1969). *Conjectures and Refutations,* Routledge and Kegan Paul, London.

Tarski, A. (1952). The semantic conception of truth and the foundations of semantics, in *Semantics and the Philosophy of Language,* L. Linsky ed., University of Illinois Press, Urbana, Illinois.